Multivariable GA-Based Identification of TS Fuzzy Models: MIMO Distillation Column Model Case Study

Borhan Molazem Sanandaji, Karim Salahshoor, and Alireza Fatehi

Abstract—**In this paper, a nonlinear fuzzy identification approach based on Genetic Algorithm (GA) and Takagi-Sugeno (TS) fuzzy system is presented for fuzzy modeling of a multi-input, multi- output (MIMO) dynamical system. In this approach, GA is used for tuning the parameters of the membership functions of the antecedent parts of IF-THEN rules and Recursive Least-Squares (RLS) algorithm is employed for parameter estimation of the consequent linear sub-model parts of the TS fuzzy rules. The presented method is implemented on a simulated nonlinear MIMO distillation column. The results show that the presented method gives a more accurate model in comparison with the conventional TS fuzzy identification approach.**

I. INTRODUCTION

Fuzzy model identification is an effective tool for the approximation of nonlinear dynamical systems on the basis of measured data [2]. This approach has been popular for the past years due to its ability to utilize heuristic knowledge to provide quantitative model which can accurately represent complex nonlinear systems.

Among different fuzzy modeling techniques, Takagi-Sugeno (TS) model [1] has received a great deal of attention and has been employed in many applications in nonlinear system identification. This is mainly due to its good results in different applications and also because it employs mathematical functions as rule consequent parts.

This model consists of IF-THEN rules with fuzzy antecedents and mathematical functions in the consequent parts. This structure gives the ability to utilize the input-output data in an efficient way.

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The construction of a TS model is usually done in two steps. In the first step, the fuzzy sets (membership functions) in the rule antecedent parts are determined. This can be done manually, using knowledge of the process, or by some data-driven techniques. In the second step, the parameters of the consequent functions are estimated. The bottleneck of the construction procedure is the identification of the antecedent membership functions, which is a nonlinear optimization problem [15]. Because the model obtained from the TS method is dependent on the membership functions, the choice of the fuzzy sets will affect the accuracy of the model. Therefore, one of the challenges of improving the accuracy of the fuzzy model is to tune the fuzzy sets such that the mean squared error between the model and the actual system is minimized [6].

In this paper, Genetic Algorithm (GA) is used for parameter identification and tuning of the membership functions [3],[6].Recursive-Least Squares (RLS) algorithm is used for parameter estimation of the output equations in the consequent parts [4].In addition, the presented approach has been developed for MIMO TS fuzzy models [12],[13].

This technique is applied to a simulated nonlinear distillation column as a popular benchmark problem in nonlinear identification [5], [7], [10].The results are compared with the results obtained from the conventional TS fuzzy model approach. The paper is organized as follows. In section II, the MIMO TS fuzzy model is introduced. In section III, the RLS method is presented. The GA-based method is described in section IV. The distillation column model is given in section V and finally the presented technique is applied to the distillation column in section VI. Some brief conclusion remarks are given in section VII.

II. MIMO TS FUZZY MODEL

In the TS fuzzy model, input variables are quantified by the means of linguistic values using membership functions as in standard fuzzy systems [6]. The major privilege of the TS fuzzy model is that it uses mathematical functions in the consequent parts instead of using fuzzy sets. The structure can be seen as a combination of linguistic and mathematical regression modeling [6]. In this paper, a MIMO extension of the TS fuzzy model is considered [12], [13]. Therefore, the i^{th} rule of the MIMO TS fuzzy model has the following form:

R^i :

$IF\ x_1\ is\ A_1^i\ AND\ x_2\ is\ A_2^i\ AND...AND\ x_n\ is\ A_n^i$
$THEN$

$$y^i = X_e^T \pi^i \quad \text{i=1,2,…,M} \tag{1}$$

where,

$x_1, x_2, ..., x_n$ are input variables;

n is the total number of input variables;

$y^i = [y_1^i, y_2^i, ..., y_m^i]$ is the output of i^{th} rule;

m is the total number of output variables;

M is the total number of rules;

$A_j^i (A_1^1 ... A_n^M)$ are the linguistic terms of input fuzzy sets;

In (1), R^i denotes the i^{th} fuzzy rule; X_e^T is the extended input vector, $X_e^T = [1, X^T]$, which is formed by appending the input vector $X = [x_1, x_2, ..., x_n]^T$ with 1 to define a free bias parameter for each rule [11],[13].

As a MIMO extension of the TS model, y^i in the consequent part of the fuzzy rule denotes the multidimensional vector of the i^{th} linear sub-system [13]. Similarly, the parameters in the consequent parts will be denoted by:

$$\pi^i = \begin{bmatrix} a_{01}^i & a_{02}^i & \dots & a_{0m}^i \\ a_{11}^i & a_{12}^i & \dots & a_{1m}^i \\ \dots & \dots & \dots & \dots \\ a_{n1}^i & a_{n2}^i & \dots & a_{nm}^i \end{bmatrix} \tag{2}$$

In order to calculate the crisp output for the fuzzifier, Mamdani MIN operator can be used, and defuzzifiction may be obtained using the weighted average method as follows:

$$y = \frac{\sum_{i=1}^{M} y^i \alpha^i}{\sum_{i=1}^{M} \alpha^i} \quad \text{(Crisp Output)} \tag{3}$$

where

$$\alpha^i = \prod_{j=1}^{n} \mu_{A_j^i}(x_j) \tag{4}$$

α^i is the fulfillment degree of the antecedent part for the i^{th} rule.

Equation (3) can be rewritten as follows:
Defining:

$$\xi^i = \frac{\alpha^i}{\sum_{i=1}^{M} \alpha^i} \tag{5}$$

Equation (3) becomes:

$$y = \sum_{i=1}^{M} y^i \xi^i \tag{6}$$

Then, by substituting (1) in (6), it gives:

$$y = \sum_{i=1}^{M} \left(X_e^T \pi^i \right) \xi^i \tag{7}$$

III. Recursive-Least Squares Algorithm

Recursive-Least Squares (RLS) algorithm is used to estimate the parameters in matrix π^i, defined in (2), which is dependent on the values of the membership functions. In order to use RLS identification algorithm, (7) can be expanded as follows:

$$y = \xi^1 X_e^T \pi^1 + \xi^2 X_e^T \pi^2 + ... + \xi^M X_e^T \pi^M \tag{8}$$

which can be defined in the form of two new terms φ and θ as :

$$\varphi = [\xi^1 X_e^T, \xi^2 X_e^T, ..., \xi^M X_e^T]^T \tag{9}$$

$$\theta = \left[(\pi^1)^T, (\pi^2)^T, ..., (\pi^M)^T \right]^T \tag{10}$$

Consequently, (7) can be rewritten as follows:

$$y = \varphi^T \theta \tag{11}$$

The RLS algorithm can then be conducted in the following steps [8], [13]:

1) Form the new φ_k^T at each sample time and then evaluate the following gain vector named as Kalman gain vector:

$$K_k = C_k \varphi_k = \frac{C_{k-1} \varphi_k}{1 + \varphi_k^T C_{k-1} \varphi_k} \tag{12}$$

2) Update the parameter matrix θ_k :

$$\theta_k = \theta_{k-1} + K_k e_k \tag{13}$$

where

$$e_k = y_k - \varphi_k^T \theta_{k-1}$$

3) Update the covariance matrix C_k :

$$C_k = C_{k-1} - \frac{C_{k-1} \varphi_k \varphi_k^T C_{k-1}}{1 + \varphi_k^T C_{k-1} \varphi_k} = \left[\mathrm{I} - K_k \varphi_k^T \right] C_{k-1} \tag{14}$$

where $\theta_1 = \left[(\pi^1)^T, (\pi^2)^T, ..., (\pi^M)^T \right]^T = 0; C_1 = \Omega \mathrm{I}$

4) Repeat the procedure from step one for the next data sample.

IV. GENETIC ALGORITHM (GA)

In this paper, GA is used to tune the parameters of the input fuzzy sets in the antecedent parts [3], [6]. GA will search for the best configuration of input fuzzy sets for each input variable based on the fitness function which in this case is the mean of squared error (MSE) between the fuzzy model output and the actual output given by [6]:

$$MSE = \frac{\sum_{i=1}^{N}\left(y_k - \hat{y}_k\right)^2}{N} \quad (15)$$

where N is the total number of training data samples, y_k is the actual output at the sample time k and $\hat{y}_k$ is the estimated output at the sample time k.

Membership functions are assumed to be as symmetrical Gaussian function:

$$\mu_j^i(x_j) = \exp\left(-\frac{1}{2}\left(\frac{x_j - c_j^i}{w_j^i}\right)^2\right) \quad (16)$$

One step to encode the whole fuzzy sets is to directly encode the values of c_j^i (center of the i^{th} membership function) and w_j^i (width of the i^{th} membership function) as they are in the universe of discourse. But, this would require imposing some constraints on chromosome variables to ensure that w_j^i gets positive values and c_j^i is monotonically increasing because it represent centers of fuzzy membership functions such as small (c^1), medium (c^2) and large (c^3). For instance, if c^2 is bigger than c^3 with a certain gap for a given input, it will leave a space where membership value for that space cannot be defined [6]. In order to incorporate these constraints into the GA, each center can be coded as the positive distance from the previous center. This ensures that during the whole optimization procedure small is less than medium, medium is less than large, etc. Therefore, a good alternative is to encode the distance between two c_j^i as shown below:

c^1	w^1	$c^2 - c^1$	w^2	$c^3 - c^2$	w^3

Fig .1 Sample Chromosome.

where the index of c or w is the number of the related fuzzy set for the selected input. For example, c^3 means parameter c of the third fuzzy set for the selected input. By this way of chromosome definition, it will be assured that all of the chromosome variables are positive.

Once the encoding of the chromosomes has been determined, the automatic tuning algorithm of the input fuzzy sets using GA is conducted as follows [6]:

1) A string is defined with a necessary length to represent the parameters of the whole fuzzy sets for all inputs.
2) A random initial population is made.
3) MSE is calculated for each chromosome.
   a) Considering fixed parameters for fuzzy sets, RLS is used to obtain the parameters of the consequent functions.
   b) Calculate MSE for each chromosome.
4) Selection (Roulette function), crossover (Scattered) and mutation (Gaussian) operators are performed to produce new population.
5) Steps 3 and 4 are repeated until a stopping criterion that sets by user is achieved to get the best chromosome. In this paper, maximum number of generations is selected for this criterion.
6) Using the best chromosome, the best configuration of the input fuzzy sets will be obtained in terms of the optimal values of c_j^i and w_j^i.

V. DISTILLATION COLUMN CASE STUDY

A. Process Overview

The process to be identified is a first-principle model of a binary distillation column (see Figure 2).The column is referred to as "column A" which has been studied in several papers [10]. The column has 39 trays, a reboiler and a condenser. The modeling assumptions are equilibrium on all trays, total condenser, constant molar flows, no vapor holdup, and linearized liquid dynamic. The simulated process covers the most important effects for the dynamic of a real distillation column. The model is a 4x4 "open-loop" (uncontrolled) column [10] with four manipulated variables (boilup flow rate, V, reflux flow rate, L, distillate product flow rate, D, bottom product flow rate, B) and four controlled variables (bottom product composition, x_B, top product composition, y_D, reboiler holdup, M_B, condenser holdup, M_D). Further details of the simulated process model are described in [5], [7], and [10].

To identify the simulated process, an appropriate model should be selected. It is assumed that the process under study can be represented by the following one-step prediction model [5], [14]:

$$\hat{y}(k) = f[y(k-1), y(k-2), u(k-1)] \quad (17)$$

where $y(k)$ and $u(k)$ are the process output and input vectors at time k, respectively. This structure is selected due to its intrinsic nonlinear characteristics and its generality.

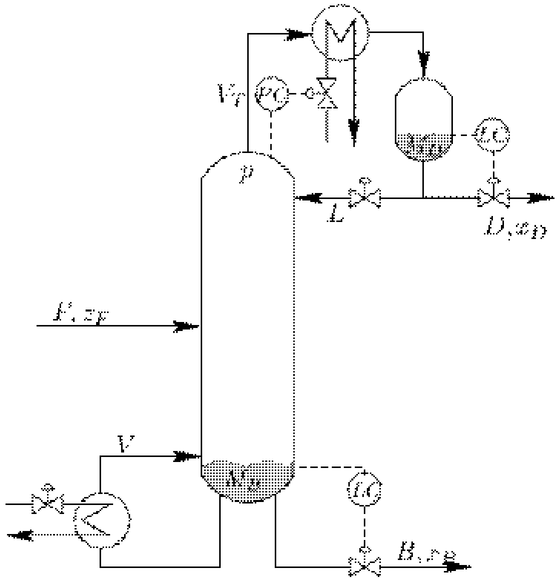

Fig.2 Distillation Column

In the identification procedure, L and V are used as inputs while M_B and M_D are used as outputs.

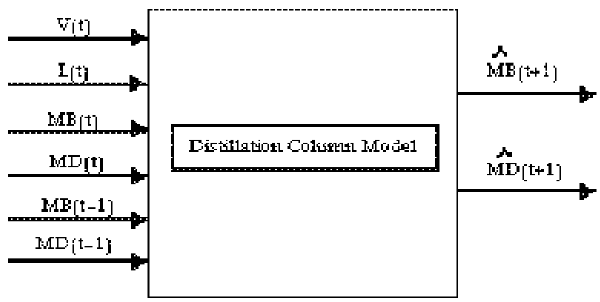

Fig.3 Block diagram of distillation column fuzzy model.

Figure 3 shows the block diagram of the assumed input-output variables for the fuzzy model identification of the simulated distillation column. As shown, based on (17), there are 6 inputs and 2 outputs for the considered identification scheme.

B. *Identification Test*

Generalized Binary Noise (GBN) is used as the input test signal [9] .GBN is suitable for control-relevant identification of industrial processes. The character of a GBN signal can be determined by its average switching time and its amplitude [9]. The white noise GBN is obtained when the average switching time is twice the minimum switching time. The amplitudes of GBN signals can be determined by a priori knowledge of the process .They are chosen in to generate rich data with good enough signal to noise ratio without disturbing the product quality. The steady state values of the proposed distillation column are given in [10].These values are used for the process initial conditions. The GBN signals which have been applied as input changes in boil up flow V and reflux flow L are depicted in Figures 4 and 5, respectively.

VI. Application to Distillation Column

From 600 samples of data, 300 are used for identification of fuzzy parameters and the rest will be used for evaluation purposes. As indicated in Figure 3, there are 6 inputs to the distillation model. For each of the inputs, two fuzzy membership functions will be considered as LOW and HIGH. Thus, the rule-base will consist of 64 rules in the consequent parts. As an example, the i^{th} rule is demonstrated in the following:

$$
\begin{aligned}
&R^i:\\
&IF\ V(t)\ is\ A_1^i\ AND\ L(t)\ is\ A_2^i\ AND\\
&M_B(t)\ is\ A_3^i\ AND\ M_D(t)\ is\ A_4^i\ AND\\
&M_B(t-1)\ is\ A_5^i\ AND\ M_D(t-1)\ is\ A_6^i\\
&THEN
\end{aligned}
$$

$$
\begin{aligned}
M_B(t+1)^i &= a_{01}^i + a_{11}^i V(t) + a_{21}^i L(t) + a_{31}^i M_B(t)\\
&+ a_{41}^i M_D(t) + a_{51}^i M_B(t-1) + a_{61}^i M_D(t-1)
\end{aligned}
$$

$$
\begin{aligned}
M_D(t+1)^i &= a_{02}^i + a_{12}^i V(t) + a_{22}^i L(t) + a_{32}^i M_B(t)\\
&+ a_{42}^i M_D(t) + a_{52}^i M_B(t-1) + a_{62}^i M_D(t-1)
\end{aligned}
$$

Using Mamdani MIN operation, the membership value for each input will be obtained. Then, using the weighted average as defuzzification operation, the output can be calculated [6]. The values will be used by RLS to estimate the best values of the parameters a_{nm}^i. Several experiments have been performed to test the effectiveness of the identified fuzzy model with automatically tuned fuzzy sets using GA with the following settings:

- Number of chromosome variables:24
- Number of populations:20
- Number of generations:100
- Crossover: Scattered
- Mutation function: Gaussian
- Selection function :Roulette
- Elite count:5
- Crossover fraction:0.8

The results have been compared with the conventional TS fuzzy model. For comparison purposes, several simulation tests have been conducted by trial and error procedure to obtain the best possible fuzzy sets for the conventional TS identification.

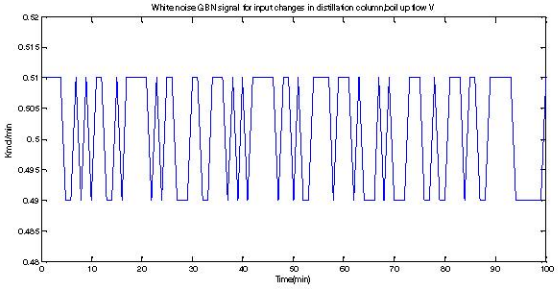

Fig.4 GBN signal for input changes, boil up flow V

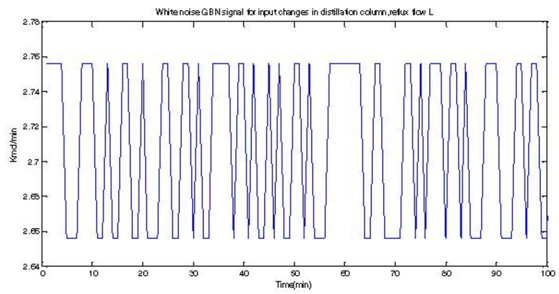

Fig.5 GBN signal for input changes, reflux flow L

In Figures 6 and 7, the corresponding outputs M_B and M_D estimated by the conventional TS fuzzy model and the presented GA-based approach are shown. The mathematical model represents the simulated distillation column from which the data has been obtained. Table I shows the evaluation results in terms of the MSE for each model.

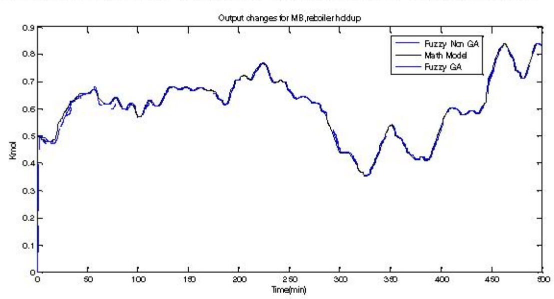

Fig.6 Output changes for MB (t), reboiler holdup

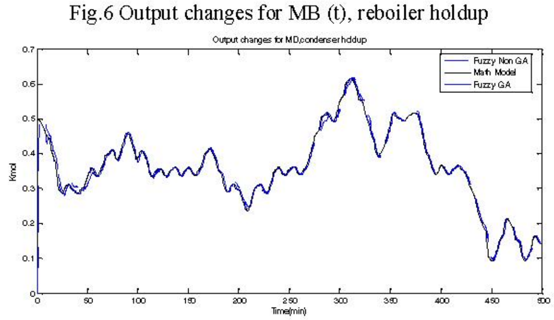

Fig.7 Output changes for MD (t), condenser holdup

It can be seen that the GA-based fuzzy model gives a more accurate model in comparison with the conventional TS fuzzy model. Figures 8 and 9 show the membership functions for V and L being tuned by the presented GA-based method.

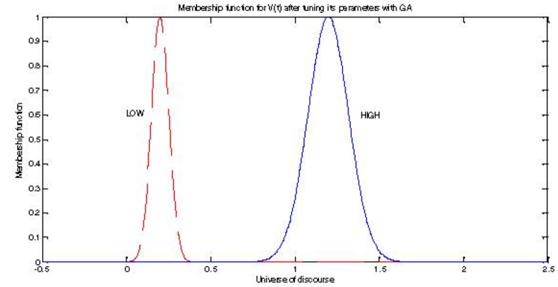

Fig.8 Membership functions for V (t) after tuning their parameters by GA.

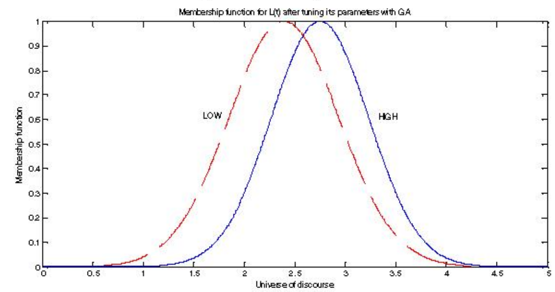

Fig.9 Membership functions for L (t) after tuning their parameters by GA.

TABLE I

MEAN SQUARE ERROR COMPARISON FOR OUTPUTS

Model	MSE
With GA	3.89 x 10^-1
Without GA	7.72 x 10^-1

VII. CONCLUSION

In this paper, a MIMO GA-based TS fuzzy identification approach has been presented which uses GA to automatically tune the parameters of fuzzy membership functions. Evaluating the simulation results, it has been shown that the presented method can estimate a more accurate MIMO fuzzy model for the distillation column benchmark problem in comparison with the conventional TS fuzzy identification approach. At the time of preparing this paper, our main focus was on the functional comparison of the presented approach with the conventional TS fuzzy identification approach. It will be our next research aim to compare the presented approach with other well-known fuzzy identification approaches such as ANFIS. This work will be carried out in the near future and the results will be published.

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